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SOLUTIONS OF PROBLEMS IN NUMBER FOUR.

SOLUTIONS of problems in No. 4 have been received as follows:

From Prof. A. B. Evans, 444; W. E. Heal, 444; Henry Heaton, 444; Prof. A. Hall, 444; William Hoover, 441; E. H. Moore, Jr., 441, 444; P. Richardson, 442; Prof. O. Root, Jr., 444; Prof, E. B. Seitz, 444; Thomas Spencer, 441.

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441. *By Wm. Hoover, A. M., Dayton, Ohio.*—"A cone revolves around its axis with a known angular velocity. The altitude begins to diminish and the vertical angle to increase, the volume being constant. Show that the angular velocity is proportional to the altitude."

SOLUTION BY THE PROPOSER.

Let ω, ω_1 be the known and required angular velocities, r, r_1 the corresponding radii of base, h, h_1 the altitudes, k, k_1 the radii of gyration, and m the constant mass.

The moment of the momentum being constant,

$$\tfrac{1}{2}mk^2\omega = \tfrac{1}{2}mk_1{}^2\omega_1;$$

but

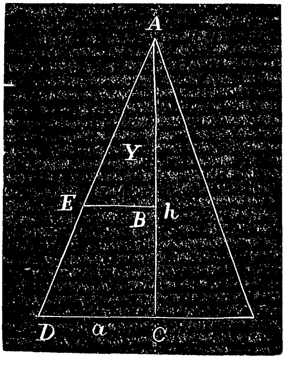
$$k^2 = \tfrac{3}{10}r^2 = \frac{9m}{10\pi h}, \quad k_1{}^2 = \tfrac{3}{10}r_1{}^2 = \frac{9m}{10\pi h_1}.$$

Substituting,

$$\omega_1 = \frac{h_1}{h}\omega.$$

SOLUTION BY THOMAS SPENCER, SOUTH MERIDEN, CONN.

As there are no external forces, the moment of momentum of the cone about its axis will be constant. Let us call this M, then we have



$$M = m\int_0^h\int_0^{\frac{ax}{h}}\int_0^{2\pi}\omega r^3 dx dr d\theta = \tfrac{1}{10}m\pi\omega a^4 h,$$

where m is the mass of a unit of volume, h = the height AC, a the radius of the base, $x = AB$, ra = radius of gyration, θ = the vertical angle and ω the given angular velocity.

But the volume of the cone is constant;

$$\therefore \frac{\pi a^2 h}{3} = V, \text{ a constant; which gives}$$

$$a^4 = \frac{9V^2}{\pi^2 h^2}.$$

Substituting this value in the above, we have

$$\omega = \frac{10M\pi}{9mV^2} \cdot h.$$

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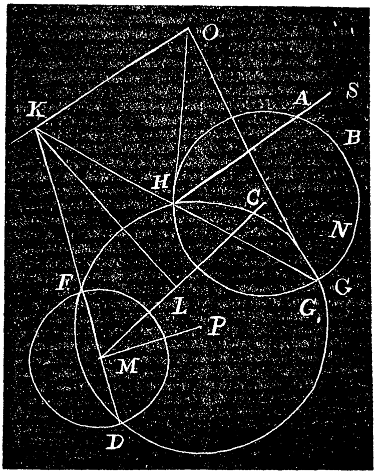
442. *By Prof. Casey.*—"*ABN* is a given circle, *D, F* and *O* are given points in the same plane. It is required to describe a circle passing through *D* and *F* and intersecting the given circle in the points *G, H,* so that the triangle *GOH* may be of a given magnitude."

SOLUTION BY P. RICHARDSON, BROOKLYN, N. Y.

Let *C* be the center of the given circle *ABN; D, F* and *O* the given points. Bisect *DF* in *M;* with radius *MD* and center *M* describe a circle. Join *CM* and draw the radical axis, *LK,* of the circles *ABN* and *DF.* Produce *DF* to meet the radical axis *LK* in *K,* then *K* is a given point; join *KO,* it is a given line. Draw *SH,* parallel to *OK* and at a given distance from it, intersecting the given circle in *H.* Join *KH* and produce it to meet the circle *ABN* in *G.* Now *O, H* and *G* are given points, hence the sides of the triangle *OHG* are given in magnitude. With center *P* describe a circle through *DFH* and let *KH* produced meet this circle in G_1. It now remains to show that *G* and G_1 are coincident points.



Because *K* is a point in the radical axis *LK,* hence $DK \times FK = HK \times GK$; but *DF* is the radical axis of circles *DF* and *DFH,* hence we have $DK \times FK = HK \times G'K$; therefore *G* and *G'* are the same point.

COR. If a tangent *KR* be drawn to circle *ABN* the circle through *D, F* and *R* will be tangent to *ABN.*

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443. *By O. H. Merrill.*—"In cutting the maximum rectangular parallelopipedon from a frustum of a cone, five pieces are cut off. Find the volume of each of these pieces."

No solution received.

[Putting a for the side of a square inscribed in the lower base, b for the side of a square inscribed in the upper base, h for the height of the frustum, and x for the height of the maximum parallelopipedon, we find for the volume V_1, correponding to the height x,

$$V_1 = \frac{a^2h^2x - 2ahcx^2 + c^2x^3}{h^2},$$

where $c = a - b$. Differentiating for a maximum and solving for x we find

$$x_1 = \frac{ah}{3(a-b)}.$$

The height of the maximum parallelcpipedon will therefore be x_1 and the area of its horizontal faces will be $\frac{4}{9}a^2$, and therefore its volume, V_1, $= \frac{4}{9}a^2x_1$. This subtracted from the volume of the frustum V'' whose h'ht is x_1 will give the volume, $2V_2 + 2V_3$, of the four slabs cut off by the four vertical planes which coincide with the four vertical sides of the maximum parallelopipedon. If the vertical sections are made simultaneously, the volume cut off will consist of eight pieces, four of which will be respectively eq'l to V_3, and the other four pieces, $4V_4$, will also be similar and equal, each to each; their vertices will be points, their altitude x_1, and the sum of the areas of their bases may be expressed in values of a, b, h and π and is therefore a known quantity; call this sum m, then $V_4 = \frac{1}{12}mx_1$, and $2V_2 = 2V_3 + \frac{1}{3}mx_1$. The volumes of the pieces cut off will therefore be :—

1. A frustum V', whose height is $h - x_1$ and the radii of whose bases are r and $\frac{2}{3}R$, r and R being the radii of the bases of the given frustum.

2 and 3. Two equal slabs, each $= V_2 = \frac{1}{4}(V'' - V_1 - \frac{1}{3}mx_1) + \frac{1}{6}mx_1$; where V'' is a frustum whose height is x_1 and radii of bases R and $\frac{2}{3}R$.

4 and 5. Two equal slabs, each $= V_3 = \frac{1}{4}(V'' - V_1 - \frac{1}{3}mx_1)$.

If b is equal to, or greater than, $\frac{2}{3}a$, $V' = 0$, and V'', in the values of 2, 3, 4 and 5, becomes V, the volume of the given frustum.]

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444. *Selected by Prof. H. T. Eddy.*—"Given the five equations,

$$\begin{aligned} x_1^2 + x_2^2 + x_3^2 &= 3\beta^2, \\ y_1^2 + y_2^2 + y_3^2 &= 3a^2, \\ x_1y_1 + x_2y_2 + x_3y_3 &= 0, \\ x_1 + x_2 + x_3 &= 0, \\ y_1 + y_2 + y_3 &= 0. \end{aligned}$$

Eliminate x_2y_2 x_3y_3, and show that

$$a^2x_1^2 + \beta^2y_1^2 = 2a^2\beta^2.$$

(Routh's Dynamics, 4th Edition, Article 38.)"

SOLUTION BY PROF. ASAPH. HALL.

Problem 444 is a simple piece of elimination. The values of x_3 and y_3 from the two last equations give

$$x_1^2 + x_2^2 + x_1x_2 = \tfrac{3}{2}\beta^2,$$
$$y_1^2 + y_2^2 + y_1y_2 = \tfrac{3}{2}a^2,$$
$$2x_1y_1+2x_2y_2+x_1y_2+x_2y_1 = 0.$$

The two equations give

$$x_2 = -\tfrac{1}{2}x_1^2 \pm \surd(\tfrac{3}{2}\beta^2 - \tfrac{3}{4}x_1^2),$$
$$y_2 = -\tfrac{1}{2}y_1^2 \pm \surd(\tfrac{3}{2}a^2 - \tfrac{3}{4}y_1^2).$$

Substituting these values of x_2, y_2 in the third equation and reducing we have

$$a^2x_1^2 + \beta^2y_1^2 = 2a^2\beta^2.$$

SOLUTION BY PROF. ASHER B. EVANS.

Put $x_2 = nx_1$ and $y_2 = my_1$; then, from equations fourth and fifth,

$$x_3 = -(n+1)x_1, \qquad y_3 = -(m+1)y_1.$$

By aid of these values of x_2 y_2 x_3 y_3 we may reduce equation third to

$$x_1y_1(2+2mn+m+n) = 0.$$

Place the factor $(2+2mn+m+n)$ equal to zero, then

$$m = -\frac{n+2}{2n+1},$$
$$-(m+1) = \frac{1-n}{2n+1}.$$

These values enable us to reduce equations first and second to the forms

$$2x_1^2(1+n+n^2) = 3\beta^2,$$
$$2y_1^2(1+n+n^2) = a^2(2n+1)^2.$$
$$\therefore 2a^2x_1^2(1+n+n^2) = 3a^2\beta^2,$$
$$2\beta^2y_1^2(1+n+n^2) = a^2\beta^2(2n+1)^2.$$

By addition

$$2(a^2x_1^2+\beta^2y_1^2)(1+n+n^2) = 4a^2\beta^2(1+n+n^2).$$
$$\therefore a^2x_1^2 + \beta^2y_1^2 = 2a^2\beta^2.$$

SOLUTION BY HENRY HEATON, ATLANTIC, IOWA.

From equations (1), (2), (4) and (5), we see that x_1, x_2, x_3 are the roots of a cubic equation in which the coefficient of x^2 is zero, and that of x is

$-\frac{3}{2}\beta^2$; and that y_2, y_2, y_3 are the roots of another in which the coefficient of y^2 is zero and that of y is $-\frac{3}{2}a^2$. Hence if

$$x_1 = \sqrt{2}.\beta \cos \varphi,$$
$$x_2 = \sqrt{2}.\beta \cos (\varphi + \tfrac{2}{3}\pi),$$
$$x_3 = \sqrt{2}.\beta \cos (\varphi + \tfrac{4}{3}\pi);$$
$$y_1 = \sqrt{2}.a \cos \theta,$$
$$y_2 = \sqrt{2}.a \cos (\theta + \tfrac{2}{3}\pi),$$
$$y_3 = \sqrt{2}.a \cos (\theta + \tfrac{4}{3}\pi).$$

Equation (3) of the problem then becomes

$$2a\beta[\cos\theta\cos\varphi + \cos(\theta+\tfrac{2}{3}\pi)\cos(\varphi+\tfrac{2}{3}\pi) + \cos(\theta+\tfrac{4}{3}\pi)\cos(\varphi+\tfrac{4}{3}\pi)] = 0.$$

But twice the product of the cosines of two arcs is equal to the cosine of the sum of the two arcs plus the cosine of their difference. Hence

$$3\cos(\theta-\varphi) + \cos(\theta+\varphi) + \cos(\theta+\varphi+\tfrac{4}{3}\pi) + \cos(\theta+\varphi+\tfrac{2}{3}\pi) = 0.$$

But $\cos(\theta+\varphi) + \cos(\theta+\varphi+\frac{2}{3}\pi) + \cos(\theta+\varphi+\frac{4}{3}\pi) = 0$ whatever the values of θ and φ; $\therefore \cos(\theta-\varphi) = 0$, and $\theta-\varphi = \frac{1}{2}\pi$. Hence

$$\cos^2\theta + \cos^2\varphi = 1.$$

But $\cos\theta = \dfrac{y_1}{\sqrt{2}.a}$ and $\cos\varphi = \dfrac{x_1}{\sqrt{2}.\beta}$. Therefore

$$\frac{y_1^2}{2a^2} + \frac{x_1^2}{2\beta^2} = 1;$$
$$\therefore a^2x_1^2 + \beta^2y_1^2 = 2a^2\beta^2.$$

Since $\theta-\varphi = \frac{1}{2}\pi$, $\cos^2(\theta+\frac{2}{3}\pi) + \cos^2(\varphi+\frac{2}{3}\pi) = 1$. Hence

$$a^2x_2^2 + \beta^2y_2^2 = 2a^2\beta^2,$$

and in a similar manner it may be shown that

$$a^2x_3^2 + \beta^2y_3^2 = 2a^2\beta^2.$$

CORRESPONDENCE.

Editor Analyst:

I was somewhat surprised that you should think that your readers would not take much interest in what I regard the most curious mathematical discoveries ever made, and which have enabled me to calculate the logarithms of all the prime numbers under 100 000 true to 40 decimal places, and all under 3000 true to 64 places; and all the Napierian logarithms under 1300 true to 76 places. Also to make four sets of tables by either of which I can find the logarithm of any number whatever, true to 65 decimal places and